Superconvergence Relations and Parity Violating analogue of GDH sum rule.

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Sum rules of superconvergence type for parity violating amplitudes (p.v. analogue of Gerasimov-Drell-Hearn sum rule) are considered. Elementary processes initiated by polarized photons in the lowest order of electroweak theory are calculated as examples illustrating the validity of the p.v.sum rules. The parity violating polarized photon-induced processes for proton target are considered in the frame of effective low energy theories and phenomenological models based on p.v. nucleon-meson effective interactions. Assuming the saturation of p.v. sum rule the possibility to limit the range of the parameters, poorly known from existing experimental data and used in these models is discussed. The asymmetries for p.v. π^0 and π^+ production, measurable in future high intensity polarized photon beams experiments, are given.

I. INTRODUCTION

The GDH sum rule [1] is recently intensively measured [2, 3] and considered as a clean and important test of the knowledge of the nucleon spin structure especially in the resonance region [4]. The rising interest in GDH (and its Q^2 dependence generalization) and more generally in spin structure of nucleon has started with the new generation of precise spin experiments [5, 6]. First direct data for real photons taken at MAMI [3]are especially important in understanding the subject and new data also with higher energies are now available and expected in future from ELSA, GRAAL, Jlab and Spring-8 [7, 8, 9]. The experiments based on intense polarized beams of photons [3, 10] give also the opportunity to test the weak (parity violating) part of photon-hadron interactions. The knowledge of p.v. couplings in nucleon-meson and nucleon-nucleon forces is a very important point for understanding physics of nonleptonic weak p.v. hadronic interactions. In addition the $\gamma\rho\pi$ and $\gamma\Delta N$ p.v. couplings, very poorly known, can also play role in photon-induced reactions.

It was shown in the paper [11] that the polarized photon asymmetry in π^+ photo-production near the threshold could be a good candidate to measure p.v. pion-nucleon coupling h_π^1. Similar expectations are connected with low energy Compton scattering [12, 13]. Let us mention here that the h_π^1 coupling has been measured in nuclear [14] and atomic [15] systems. The extraction of h_π^1 from such experiments is however difficult due to poor understanding of many-body systems. In fact, the disagreement between ^{18}F and ^{133}Cs experiments is seen [14, 15].

The experimental observation of p.v. effects in photonic reactions is generally difficult because the expected asymmetries are very small. However, it seems sound to expect that the new high luminosity machines, generating intense polarized photon beams can change the situation in the nearest future [11, 16]. Having this in mind a set of sum rules for parity violating part of Compton amplitudes has been recently derived by one of us (L.Ł) [17]. In particular, p.v. analogue of GDH sum rule, based on Low Energy Theorems [18] and under assumption of superconvergence of the type $\frac{f(z)}{z} \to 0$ at infinity for asymmetric amplitude was formulated there. In the past a number of superconvergence type sum rules for parity conserving Compton Scattering (one example of which reduces to GDH sum rule) has been obtained and discussed [19, 20, 21] and the superconvergence relations have been also studied in detail in the very general context of axiomatic local field theory and its analyticity properties [22].

In this paper we shall discuss p.v. analogue of GDH sum rule having essentially in mind two aspects: verification and possible phenomenological implications.
The general formulae exploited in the paper are given in section 2. To verify p.v. sum rules for elementary targets we calculate in the lowest order of electroweak theory the p.v. analogue of GDH sum rule integral for the processes with polarized photons scattered off leptons (section 3). The saturation hypothesis for p.v. sum rule is discussed in section 4. In section 5 the models of p.v. low energy photon-proton interactions (i.e. heavy baryon chiral perturbation theory (HBχ PT) [13, 23] and low energy phenomenological models [24, 26, 27, 28]) are briefly described. Assuming the saturation for p.v. sum rule (similar to observed quick saturation in GDH integral) we are able to narrow down allowed values of the p.v. photon-meson and photon-Δ-nucleon couplings (poorly known) and select the models with small high energy contribution (section 6). For these selected models the energy dependence of the asymmetries for pion photoproduction is calculated (section 7) according to approach proposed in [24]. In the same section it is shown that measurement of the photon energy dependence of the asymmetries from threshold up to energy large enough to saturate

p.v. sum rule (saturation energy) allows to distinguish between the different models which obey quick saturation feature. We conclude the paper with section 8. For completeness the formulae of cross sections and asymmetries for p.v. photon-neutrino and photon-electron processes calculated in lowest order of electroweak theory are given in Appendix A. The QCD perturbative cross sections formulae for unpolarized photon scattered off polarized proton target are given in Appendix B.

II. GENERAL FORMULAE.

Let us consider p.v. Compton amplitudes for polarized photons scattered off unpolarized targets and for unpolarized photons off polarized targets. For the first case the following (crossing-antisymmetric) dispersion relation holds (compare eq.(3.18) in [17]):

$$Ref_h^{(-)\gamma} = \frac{\omega}{\pi}P\int_{\omega_{th}}^{\infty}\frac{\omega'}{\omega'^2 - \omega^2}(\sigma_h^T - \sigma_{-h}^T)d\omega' + (subtr.),$$

$$(1)$$

where ω is the photon energy in laboratory system. $f_h^{(-)\gamma}$ and σ_h^T are amplitude and relevant total cross section averaged over target particle spin, respectively; h indicates photon helicity eigenvalue.

Second possibility brings us to the following (crossing-symmetric) formulae (compare eq.(3.20) in [17]):

$$Ref_s^{(-)tg} = \frac{1}{\pi}P\int_{\omega_{th}}^{\infty}\frac{\omega'^2}{\omega'^2 - \omega^2}(\sigma_s^T - \sigma_{-s}^T)d\omega' + (subtr.).$$

$$(2)$$

Here $f_s^{(-)tg}$ and σ_s^T are amplitude and total cross section averaged over photon polarization.

It was pointed out in [17] that assumption of superconvergence for amplitude $f_h^{(-)\gamma}$ (i.e. no subtractions in (1)):

$$\frac{f_h^{(-)\gamma}(\omega)}{\omega}|_{\omega\to\infty} \to 0 \qquad (3)$$

together with Low Energy Theorem (LET) [18] leads to the p.v.analogue of GDH sum rule (once limit $\omega \to 0$ is taken in unsubtracted (1)):

$$\int_{\omega_{th}}^{\infty}\frac{\sigma_h^T - \sigma_{-h}^T}{\omega'}d\omega' = 0 \qquad (4)$$

For the polarized target case the relevant p.v. sum rule:

$$\int_{\omega_{th}}^{\infty}(\sigma_s^T - \sigma_{-s}^T)d\omega' = 0 \qquad (5)$$

can be obtained if stronger superconvergence is assumed: $f_s^{(-)tg}(\omega) \to 0$.
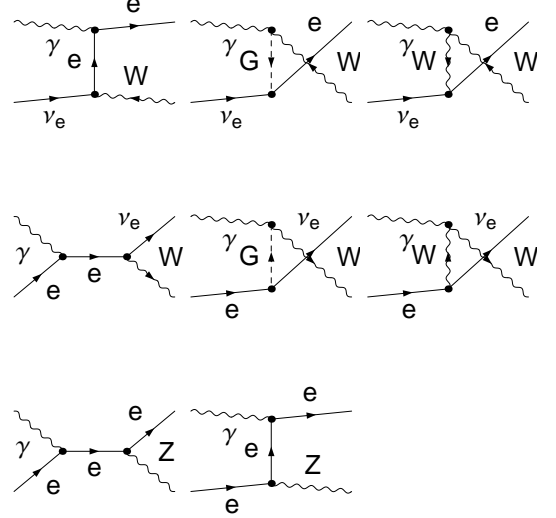

FIG. 1: The Feynman diagrams for lowest order p.v. processes induced by polarized photons

III. THE PHOTON SCATTERING OFF ELEMENTARY LEPTON TARGETS AND VERIFICATION OF P.V. SUM RULES - ILLUSTRATIVE EXAMPLES.

As examples of the elementary parity violating processes three different inelastic polarized photon scattering off lepton targets are considered:
the photon-neutrino reaction into W boson and electron, the photon-electron reactions into neutrino and W boson and into electron and Z^0 boson production. The relevant Feynmann diagrams in the lowest order of electroweak perturbation theory are depicted in fig. 1.

To be more general neutrino is treated as a massive particle in the photon-neutrino reaction. The sum rule (4) can be rewritten as follows:

$$\int_{s_{th}}^{\infty}\frac{\sigma_h^T - \sigma_{-h}^T}{(s - m^2)}ds = 0 \qquad (6)$$

where s is energy square in CM system and m is electron or neutrino mass. In contrast to nonzero contribution for GDH integral in higher order of perturbation theory the integrals in p.v. sum rules (4) and (5) should be zero at any order of perturbation theory. (More general discussion about p.c.sum rules and generalization of GDH sum rule including not only magnetic but also electron dipole moment can be found in [29].)
The calculations have been done using *FeynArts 3* package for creating amplitudes and *HighEnergyPhysics* (HEP) [30] package for calculating relevant cross sections - both written for *Mathematica* system [31].

The difference of the averaged cross sections over lepton helicity $\sigma_h^T - \sigma_{-h}^T$ in (6) is expressed by the sum of
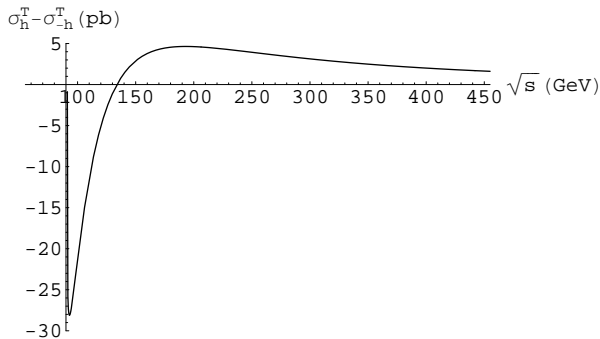
FIG. 2: The difference of the polarized photon cross sections $\sigma_h^T - \sigma_{-h}^T$ for the reaction $\gamma + e \to Z^0 + e$ as a function of energy.

$\frac{1}{2}(\Delta\sigma_+ + \Delta\sigma_-)$ where $\Delta\sigma_\pm$ are given in Appendix A, eq. (9.4). It is straightforward to verify by elementary integrations that for every considered reaction (see formulae: (9.5) and (9.6) for $\gamma\nu \to We$, (9.8) for $\gamma e \to Z^0 e$, (9.10) and (9.11) for $\gamma e \to \nu W$) the integral in eq. (6) is zero for the difference of averaged lepton cross sections.

The reaction $\gamma\nu \to We$ with massless neutrino is simplest example of the violation of the second sum rule formulated in section 2.in eq. (5). Due to the fact that only one neutrino helicity state contributes to the cross section averaged over spin of photon (unpolarized photons case) the difference of the cross sections in the integral (5) is always positive because it is simply averaged cross section of the reaction. Therefore there is no way to satisfy the condition to have integral to be zero. The interesting question is whether this sum rule could be satisfied for hadron targets when several different elementary processes might conspire to give vanishing difference of the total cross sections. We examine this possibility in the frame of perturbative QCD approach and parton language structure of photon and proton target in Appendix B.

The typical behavior of the differences of cross sections in the sum rule (4) is presented in fig. 2 for the reaction $\gamma + e \to Z^0 + e$. It is seen in the fig. 2 that the saturation of the p.v. sum rule requires high energy contribution because the approaching of the differences of total cross sections to zero is slow.

Concluding this section let us add that the photon-charged lepton reactions have been studied in the past in the frame of electroweak theory. The contribution to GDH sum rule from the processes mediated by weak bosons in the lowest order of perturbation theory has been first time discussed in [32] where also pure QED Compton scattering has been considered as well as Higgs boson production in this context. Quite recently it was shown in [33] that GDH sum rule for electron evaluated at order of α^3 agreed with the Schwinger contribution to the anomalous magnetic moment.

IV. THE SATURATION HYPOTHESIS.

One of the most interesting features of GDH sum rule for nucleon targets is a quick saturation of the GDH integral. The dominant contribution (about 90%) to the GDH sum rule proceeds from the photon's energy range from the threshold up to 0.55 GeV [2, 3, 7, 8]. The saturation hypothesis in analogy with the feature observed in the GDH sum rule is an important point in our analysis presented in section 6. Therefore we are going now to formulate the criterion of the saturation of integral in p.v. sum rule (4). It is relatively easy to define the saturation when the integral in the sum rule has non zero value as it is in the case of GDH sum rule where the value of integral is determined by anomalous magnetic moment (with electric dipole moment if generalization of GDH is considered, [29]) of target particle. The problem however appears when the integral in sum rule should be zero. Below we shall formulate the saturation criterion valid for both situations. Given any superconvergence sum rule of the form:

$$a = \int_{\omega_{th}}^{\infty} \frac{\Delta\sigma(\omega')}{\omega'} d\omega', \tag{7}$$

we define the following F quantity:

$$F(\omega) = \frac{I_0}{I_1}. \tag{8}$$

where:

$$I_0 = |a - \int_{\omega_{th}}^{\omega} \frac{\Delta\sigma(\omega')}{\omega'} d\omega'|, \tag{9}$$

and

$$I_1 = \int_{\omega_{th}}^{\omega} \frac{|\Delta\sigma(\omega')|}{\omega'} d\omega', \tag{10}$$

Requirement that $F(\omega)$ does not exceed prescribed small value at $\omega = \omega_{sat}$ determines saturation energy. The usefulness of such definition of saturation is based on the assumption that there is no large contribution to the integral from photons with energy higher than ω_{sat}. For the GDH sum rule on proton, where experimental data [3, 7] exist we can estimate ω_{sat} to be 0.55 (i.e. $\frac{E_{sat}}{E_{th}} = 1.5$ in CMS) for $F(\omega_{sat}) = 0.1$.

As there are no experimental data for p.v. sum rules on proton we shall use the values $\omega_{sat} = 0.6$ and $F((\omega_{sat}) = 0.1$ in discussion of phenomenological consequences (sections 6 and 7).

V. PROTON TARGET. THE MODELS OF P.V. LOW ENERGY PHOTON-PROTON INTERACTIONS.

In this section we shall discuss two different approaches to p.v. low energy photon-nucleon interactions. We begin with p.v. Compton amplitude on proton calculated

in the frame of HBχPT [12, 13, 23]. According to [13] the p.v. Compton amplitude can be written in CMS as follows:

$$
\begin{aligned}
M_{h_f,h_i}^{(-)s_f,s_i}(\vec{k},\vec{k'}) &= \overline{N_{s_f}}[F_1\vec{\sigma}\cdot(\hat{\vec{k}}+\hat{\vec{k'}})\vec{\epsilon_i}\cdot\overline{\vec{\epsilon_f}} \\
&\quad - F_2(\vec{\sigma}\cdot\overline{\vec{\epsilon_f}}\hat{\vec{k'}}\cdot\vec{\epsilon_i}+\vec{\sigma}\cdot\vec{\epsilon_i}\hat{\vec{k'}}\cdot\overline{\vec{\epsilon_f}}) \\
&\quad - F_3\hat{\vec{k}}\cdot\overline{\vec{\epsilon_f}}\hat{\vec{k'}}\cdot\vec{\epsilon_i}\vec{\sigma}\cdot(\hat{\vec{k}}+\hat{\vec{k'}}) \\
&\quad - iF_4\vec{\epsilon_i}\times\overline{\vec{\epsilon_f}}\cdot(\hat{\vec{k}}+\hat{\vec{k'}})]N_{s_i}, \quad (11)
\end{aligned}
$$

so that

$$
f_{\frac{1}{2}}^{(-)p} = 2F_1, \tag{12}
$$

$$
f_{h=+1}^{(-)\gamma} = -2F_4. \tag{13}
$$

To discuss p.v. sum rule and the superconvergence relations the interesting quantity is F_4 according to eq. (13). The calculations based on HBχPT analysis in NLO [13] provides value of the coefficient F_4 as follows:

$$
F_4^{NLO} = -\frac{e^2 g_A h_\pi^1 \mu_n}{8\sqrt{2}\pi^2 MF_\pi}(\omega - \frac{m_\pi^2}{\omega}arcsin^2(\frac{\omega}{m_\pi})). \tag{14}
$$

It is easy to check that at high energies the real part of $\frac{F_4^{NLO}}{\omega}$ converges to constant so superconvergence condition (3) is violated. Therefore the p.v. sum rule (4) does not hold.

The similar formula with 6 independent structure functions A_i can be written for p.c. Compton amplitude [13, 23]. For the p.c. Compton amplitude the HBχPT gives the similar results for A_3 forward scattering amplitude. According [23] A_3 is equal to:

$$
A_3^{NLO}|_{\Theta=0} = -\frac{e^2\omega\kappa_p^2}{2M^2} - \frac{e^2 g_A^2}{8\pi^2 F_\pi^2}(\omega - \frac{m_\pi^2}{\omega}arcsin^2(\frac{\omega}{m_\pi})) \tag{15}
$$

which violates superconvergence relation of the type (3) as in p.v. case.

Discussing HBχPT it is important to note the fact that the spin-dependent p.c. polarizability $\gamma_{p,n}$ (expressed by the integral similar to GDH integral but with higher power of energy in the denominator of integrand) essentially depends on loop corrections and the contribution from the Δ and the lowest order result differs not only in value but also in sign from corrected result [23, 35]. Therefore a priori it is not excluded that p.v. sum rule (4) might be satisfied if more corrections are taken into account in the frame of HBχPT. To our knowledge there is no any χPT based model for p.v. Compton amplitude which obeys the superconvergence relation (3).

Having this fact in mind we will consider existing in the literature low energy phenomenological model of pion photoproduction based on so-called pole approximation and effective Lagrangian's [24] (compare also [25]) and [26, 27, 28]. The model discussed in [24] is relevant in low energy regime so we will limit ourselves to energy below 0.55 GeV. The upper bound of energy is high enough to discuss and apply the saturation hypothesis as it was said in the previous section. The contribution from high energy region will be ignored for a moment, assuming that is unimportant.

The detailed description of the approach can be found in [24]. The asymmetries of the polarized photon cross sections for π^+ and π^0 production are expressed by the sum of the p.v. coupling constants multiplied by the relevant form factors (see figs.11-15 in [24]). In our calculations the ρNN couplings ($h_\rho^0, h_\rho^1, h_\rho^2$), ωNN couplings (h_ω^0, h_ω^1) and $\pi N\Delta$ coupling (f_Δ) have to be taken into account. For π^+ production the important contribution follows from p.v. πNN coupling (h_π^1). In addition there are two extra contributions from Δ directly coupled to photon and nucleon ($\gamma\Delta N$ coupling - μ^*) and from interaction between photon, pion and ρ meson ($\gamma\rho\pi$ coupling - h_E). The last two parameters are directly related to the p.v. photon-mesons and photon-Δ-nucleon interactions while the previous ones are related to strong sector (p.v. meson-nucleon couplings).

The knowledge of the values of p.v. couplings is rather limited; practically only ranges of values are known from experimental data (for review of the situation see [27] and references therein). On the other hand the strong sector meson-nucleon couplings can be calculated in different approaches and models reviewed in [27]; we shall exploit them in the next section. Especially difficult situation is for p.v.$\gamma\Delta N$ coupling μ^* and $\gamma\rho\pi$ (h_E) which are given in the models from [27]. Only quite large limits $\mu^* \in (-15, 15)$ and $h_E \in (-17, 17)$ in units 10^{-7} can be given for these couplings based on data analysis.

The μ^* and h_E couplings can be calculated if some extra assumptions are added. The vector-meson dominance model have been used in [24] to estimate the p.v. $\gamma\Delta N$ coupling . Neglecting ω and ϕ meson's contribution and assuming that p.v. $\rho\Delta N$ coupling is by size similar to p.v. ρNN coupling the following relation have been formulated [24]:

$$
\frac{\mu^*}{M} = \frac{h_\rho^0}{g_\rho m_\rho}. \tag{16}
$$

Taking g_ρ equal to 0.434π (after [24]) the $\mu^* \simeq 0.55h_\rho^0$ is uniquely defined by h_ρ^0 coupling.

The h_E coupling can be calculated according analysis described in [36]. Assuming so-called factorization [36] the following relation can be written for h_E:

$$
h_E = -2MG^{\rho\pi\gamma}, \tag{17}
$$

and

$$G^{\rho\pi\gamma} = \sqrt{4\pi\alpha}G^{\rho\pi}(1 \pm \frac{m_{a_1}^2 m_\rho g_T^a}{\sqrt{2}f_\rho(m_{a_1}^2 - m_\rho^2)}$$
$$\mp \frac{\sqrt{2}}{20} \frac{m_\rho g_T^b}{f_\omega(m_b^2 - m_\rho^2)}). \qquad (18)$$

Taking into account the present data [37] of the widths of a_1 and b resonances, their masses and that $G^{\rho\pi}$ is $8.9 \cdot 10^{-7}$ the two possible solutions have been found for h_E: $\sim 10 \cdot 10^{-7}$ and $\sim 1 \cdot 10^{-7}$. The results are close to the ones obtained in $SU(6)_W$ based calculations in [36].

In the next section the p.v. sum rule (4) together with saturation criterion will be used to select the models which posses quick saturation feature.

VI. PHENOMENOLOGICAL CONSEQUENCES OF SATURATION.

The p.v. meson-nucleon coupling constants calculated from the flavor-conserving part of the quark weak interactions are reviewed in [27]. The eight sets of numerical predictions for the p.v. meson-nucleon coupling constants, calculated with different assumptions and models have been summarized in Table I, taken from [27](Table 1).

Model	h_π^1	h_ρ^0	h_ρ^1	h_ρ^2	h_ω^0	h_ω^1
1. DDH, range (K=6);ref.[26]	0.0	11.4	0.0	-7.6	5.7	-1.9
2. DDH, range (K=6);ref [26]	11.4	-30.8	0.4	-11.0	-10.3	-0.8
3. DDH, ("best");ref [26]	4.6	-11.4	-0.2	-9.5	-1.9	-1.1
4. D, range (K=3);ref [28]	1.3	8.3	0.0	-8.2	-0.5	-1.8
5. D, range (K=3);ref [28]	2.0	-23.1	-0.3	-8.2	-10.6	-2.2
6. D, range (K=1);ref [28]	0.5	7.0	-0.2	-10.3	2.5	-2.0
7. D, range (K=1);ref [28]	0.4	-29.5	0.0	-10.3	-10.2	-1.1
8. KM ;[23]	0.2	-3.7	-0.1	-3.3	-6.2	-1.0

TABLE I: Predictions for the p.v. meson-nucleon coupling constants after [27]

The predictions of the p.v. couplings are grouped in five groups depending on the method of calculations (as in [27]). The strong effects are partially incorporated in the meson's and nucleon's wave functions and partially in bare quark interactions. This part of strong interaction corrections manifests themselves via dependence on K parameter (K=1 in the absence of corrections) in effective quark interactions. The first two groups called

in our notations as models 1-3 (DDH), are based on the calculations from [26]. Models from first group (1-2) contain strong corrections characterized by K=6 range parameter. The third and fourth group (models 4-5 and 6-7, D) have been calculated in [28]. The models 4 and 5 are corrected for strong interactions (K=3) while models 6 and 7 (K=1) have no strong corrections taken into account. The predictions for model 4 and 6 correspond to the factorization approximation used in the calculations while 5 and 7 are the results obtained assuming the validity of the $SU(6)_W$ symmetry. The last set of couplings (model 8, KM) is based on HBχPT calculations, [23]. All couplings are in units 10^{-7} and the $h_\rho^{1`}$ coupling, (originally listed in Table 1. from [27]) has been omitted; in fact it is zero for all models except KM. $h_\rho^{1`}$ is not used in our calculations as it does not enter in Henley et al. approach [24] .

The values of p.v. couplings listed in the table will be used to verify the quick saturation feature according the approach discussed in previous section and in [24].

The μ^* and h_E couplings will be treated as free parameters in the range limited by the experimental knowledge: $\mu^* \in (-15, 15)$ and $h_E \in (-17, 17)$ in units 10^{-7}. The contribution related to f_Δ parameter is very small in the considered approach; we have fixed this coupling to be 10^{-7} after [24].

Taking the values of p.v. coupling constants from Table I the differences of polarized photon cross sections have been calculated and used for estimation of F defined in 8. The saturation expressed by the condition $F < 0.1$ limits the allowed values of μ^* and h_E couplings.

According to [24] the most important contribution to the differences of the cross sections and asymmetries comes from πNN interaction if h_π^1 is not very small. It is expected that strong interaction corrections will increase the value of h_π^1 [27]. Indeed it is seen in Table I for D models (group 3 and 4) that increasing range K the h_π^1 also increases (models 4 and 5 with K=3 versus models 6 and 7, not corrected, K=1). On the other hand $h_\pi^1 = 0$ is also not excluded for models with higher correction range K=6 (model 1 from DDH group). The small value of h_π^1 is also supported by calculations of [23](model 8).

Apart from the model 2 and "best fit" model 3 from DDH group, quick saturation can be achieved for all other models from Table I by limiting range of free parameters μ^* and h_E. The values of μ^* and h_E obtained in (16) and in the analysis from [36] also allow to have saturation for models 4-7; the model 8 (KM) with these values of μ^* and h_E has no saturation. Experimental consequences of models with saturation are given in section 7.

It should be emphasized that the non saturating models (2 and 3) characterize large values of h_π^1. In consequence it is impossible to find any pair of μ^* and h_E couplings in allowed range to satisfy the saturation condition. This observation leads to the conclusion
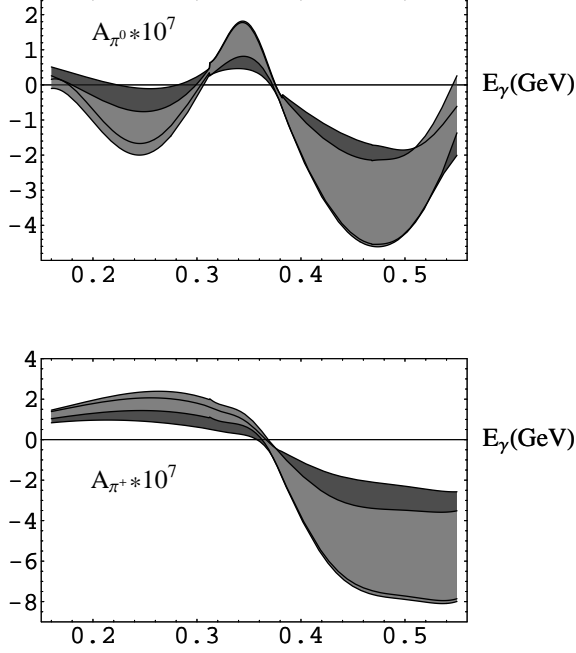
FIG. 3: The asymmetries for π^+ and π^0 photoproduction
as a function of photon energy. The shadowed bands reflect
freedom of the values of p.v. photon-meson couplings μ^* and
h_E allowed by saturation condition for model 4. The darker
band is for positive, lighter for negative values of h_E. μ^* is
always negative in shadowed bands.

that for these models some additional structure should
be observed for higher than 0.55 GeV photon's energy
(compare discussion and eq.(4.14) in ref. [17]).
Considering quick saturation as a universal feature,
related to the complexity of hadronic targets the absence
of saturation can be treated as an argument against
large value of h_π^1.

VII. PION PHOTOPRODUCTION ASYMMETRIES.

In this section we are going to discuss the experimental
consequences of the models satisfying quick saturation
i.e. the π^+ and π^0 p.v. asymmetries.

In the fig. 3 and 4 the pion photoproduction asymme-
tries are shown for strong interactions corrected models
from D groups,model 4 and 5. The π^+ asymmetries are
positive at threshold, relatively large and negative for
photon energy close to 0.5 GeV. The π^0 asymmetries on
threshold are sensitive to assumptions under which the
predictions for couplings have been calculated; the fac-
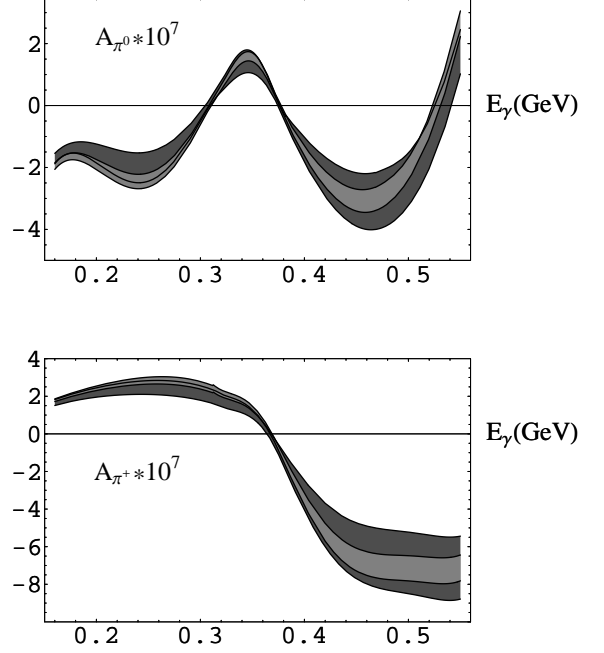torization (model 4) prefers zero or very small and rather

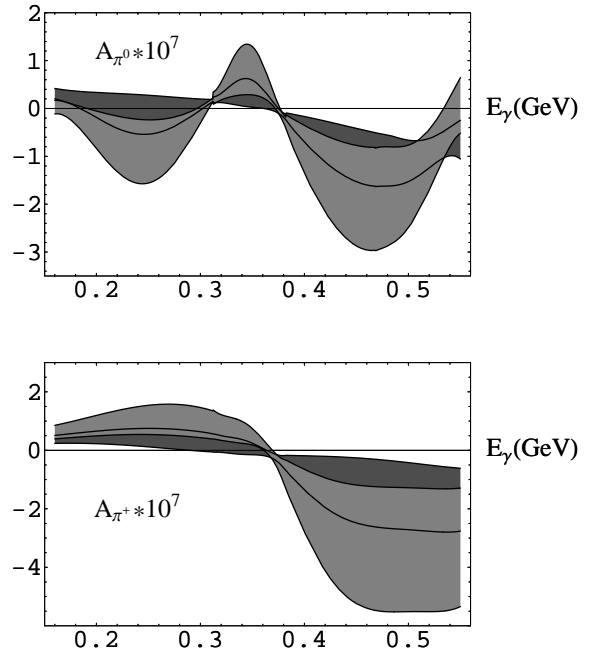FIG. 4: The asymmetries as in fig. 3 for model 5.

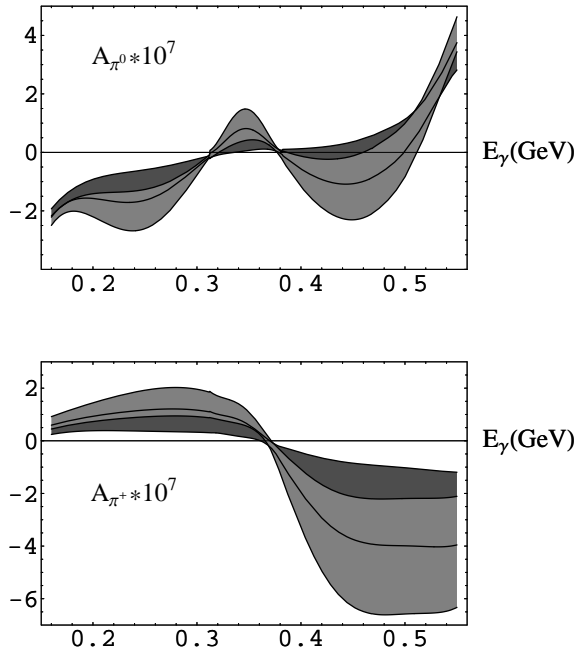FIG. 5: The asymmetries as in fig. 3 for model 6.

FIG. 6: The asymmetries as in fig. 3 for model 7.

FIG. 7: The asymmetries as in fig. 3 for model 1. Here the darker band is for positive values of μ^* and h_E couplings, lighter for negative.

positive asymmetry while $SU(6)_W$ symmetry (model 5) leads to larger and negative asymmetries.

The models 6 and 7 from D groups in which the strong interaction corrections have not been taken into account characterize smaller value of leading coupling h_π^1 and therefore the saturation is possible with much more freedom in photon-meson couplings than for corrected models 4 and 5. The asymmetries are similar in shape however smaller and less constrained (figs. 5 and 6).

Model 1 from DDH group and 8 (KM) are characterized by smallest couplings h_π^1 (0 in the case of model 1). The saturation is possible with large freedom in couplings μ^* and h_E (model 1,fig. 7). The asymmetries for model 8 (KM) are similar to those presented in fig. 7, reaching $-4*10^{-7}$ for higher energy. For the models with small or zero p.v. pion-nucleon h_π^1 the sign of μ^* and h_E couplings is related directly to the sign of π^+ asymmetry.

As a summary of this section we conclude that the most interesting models with quick saturation are models 4 and 5 from D group with corrections range K=3, based on calculations done in [28]; the predicted values of pion photoproduction asymmetries are relatively large. While the π^+ asymmetries are similar in energy dependent shape for different models, π^0 asymmetries at threshold showed the interesting model dependence (compare fig.s 3- 6)
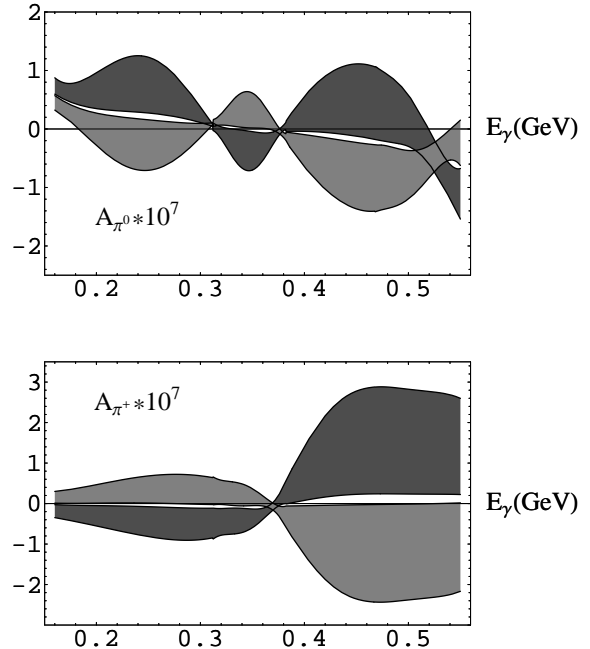
VIII. CONCLUDING REMARKS.

We have verified p.v. superconvergent sum rules formulated in [17] and we have examined their phenomenological consequences; the sum rules have been checked within the lowest perturbative order of electroweak theory for the photon induced processes with elementary lepton targets. The p.v. analogue of GHD sum rule for polarized photons interacting with unpolarized targets have been verified by straightforward calculations.
It would be interesting to check this sum rule in higher perturbative orders as it was recently done for GDH in QED in [33].
In analogy with GDH sum rule for nucleon the saturation hypothesis has been formulated and the eight models with different sets of p.v. couplings [27] have been analyzed using p.v. photoproduction model proposed in [24]. The models with largest leading p.v. pion-nucleon coupling h_π^1 do not saturate p.v. sum rule integral below energy of photon smaller than $0.6 GeV$ and the contributions from higher energies cross sections are needed. It suggests some structure in difference of the cross sections to be observed for higher photon's energy for these models.
The other models considered in the paper have enough freedom in parameter space defined by data and calculations to saturate p.v. sum rule integral below photon's

energy 0.55 GeV.

The p.v. asymmetries have been calculated in the photon energy range from threshold to 0.55 GeV. The asymmetries for some models satisfying saturation are large enough to be measured in the future experiments with intensive beams of polarized photons.

IX.   APPENDIX A.

Below the set of formulae describing polarized cross sections calculated in the lowest order of electroweak perturbative theory is presented. We consider the photon-neutrino scattering into W boson and electron, the scattering of photon-electron into neutrino and W boson and into electron and Z^0 boson.

Let us define the general formula for the cross sections as follows:

$$\sigma_n(r_1, r_2, r_3) = \frac{\pi\alpha^2}{2\sin(\Theta_W)^2 M_W^2(1-r_1)^n}[w_1^n P(r_2, r_3) \\ + w_2^n L(r_2, r_3) + w_3^n L(-r_2, r_3)]$$

where:

$$P(x,y) = \sqrt{1 + x^2 + y^2 - 2(x + y + xy)},$$
$$L(x,y) = ln(\frac{1 + x + y + P(x,y)}{1 + x + y - P(x,y)}). \qquad (19)$$

r_i are ratios $\frac{m_i^2}{s}$ and m_i are masses of particles taking part in reactions. w_i^n are coefficient functions depending on the reaction (see subsections below).

The unpolarized (averaged over photon and lepton helicity) cross sections σ and fully polarized ones σ_s^h are equal to:

$$\sigma = \frac{1}{4}(\sigma_s^h + \sigma_s^{-h} + \sigma_{-s}^h + \sigma_{-s}^{-h}) \qquad (20)$$

Here h refers to photon and s to lepton helicity ($s, h = \pm$).

The differences of the cross sections: $\Delta\sigma$ and $\Delta\sigma_s$ are defined as follows:

$$\Delta\sigma = \frac{1}{2}(\sigma_s^h + \sigma_s^{-h} - \sigma_{-s}^h - \sigma_{-s}^{-h}),$$
$$\Delta\sigma_s = \sigma_s^h - \sigma_s^{-h}. \qquad (21)$$

σ and $\Delta\sigma$ have a form of σ_3;
$\Delta\sigma_s$ is σ_2 type.

The coefficient functions w_i^3 and w_i^2 are process-dependent. We listed them in subsections.

A.   Cross sections for $\gamma\nu \to We$

The cross sections are:

$$\sigma = \sigma_3(r_\nu, r_e, r_W),$$
$$\Delta\sigma_- = \sigma_2(r_\nu, r_e, r_W),$$
$$\Delta\sigma_+ = 0$$
$$\Delta\sigma = -2\sigma. \qquad (22)$$

The coefficient functions w_i are the following:

$$w_1^2 = -4(r_\nu - r_e + 6r_W),$$
$$w_2^2 = 2(1 + r_\nu)(r_\nu - r_e) + 4r_W(1 - 2r_W - 3r_e),$$
$$w_3^2 = 4r_W(2 + r_\nu - 2r_W - 3r_e),$$
$$w_1^3 = 4(1 - r_e^2 - r_e(r_W - 2r_\nu) + 2r_W^2 - r_\nu(2 + r_W)),$$
$$w_2^3 = (1 - r_\nu)^2(r_\nu + r_e + 2r_W),$$
$$w_3^3 = 2(1 - r_e - r_W)((r_e - r_\nu)^2 + r_W(r_\nu + r_e - 2r_W)),$$
$$r_\nu = \frac{m_\nu^2}{s},$$
$$r_e = \frac{m_e^2}{s},$$
$$r_W = \frac{m_W^2}{s}. \qquad (23)$$

B.   Cross sections for $\gamma e \to Z^0 e$

The cross section are:

$$\sigma = \sigma_3(r_e, r_e, r_Z),$$
$$w_1^3 = 2r_Z((1 - \beta)r_Z - (1 + 2\beta)r_e)$$
$$\quad + \frac{1}{4}(1 - r_e)^2((1 - r_e)^2 - r_Z^2 - (1 + \beta r_Z)(1 + r_e - r_Z)),$$
$$w_2^3 = \frac{1}{2}(1 - r_e)^2 + \frac{3}{2}(1 + \beta)r_Z(1 + r_e)^2$$
$$\quad - r_Z(1 + 2\beta + 3r_e) - r_Z^2((1 - \beta)(1 - r_Z) + r_e(2 + \beta)),$$
$$w_3^3 = 0,$$
$$r_Z = \frac{m_Z^2}{s},$$
$$\beta = 4\sin(\Theta_W)^2(1 - 2\sin(\Theta_W)^2). \qquad (24)$$

and

$$\Delta\sigma_\pm = \sigma_2(r_e, r_e, r_Z),$$
$$w_1^2(+) = \frac{1}{2}(-r_e^3 + r_e^2(6 + r_Z(\beta + \delta))$$
$$\quad + r_e(7 + r_Z^2(1 - \beta - \delta) - r_Z(1 + 2\beta + 2\delta))$$
$$\quad - r_Z(5 + 3r_Z))(1 - \beta - \delta)),$$
$$w_2^2(+) = 5r_e^2 + r_e(1 - r_Z(3 - \beta - \delta))$$
$$\quad - r_Z(1 - 2r_Z)(1 - \beta - \delta),$$
$$w_3^2(+) = w_3^2(-) = 0,$$
$$w_1^2(-) = \frac{1}{2}(r_e^3 - r_e^2(6 + r_Z(\beta - \delta))$$

$$- r_e(7 + \frac{1}{2}r_Z^2(1+\delta)^2 - r_Z(1 + 2\beta - 2\delta))$$
$$+ \frac{1}{2}r_Z(5 + 3r_Z)(1+\delta)^2),$$
$$w_2^2(-) = -5r_e^2 - r_e(1 - r_Z(3 - \beta + \delta))$$
$$+ \frac{1}{2}r_Z(1 - 2r_Z)(1+\delta)^2,$$
$$\delta = 1 - 4sin(\Theta_W)^2. \tag{25}$$

$$\Delta\sigma = \sigma_3(r_e, r_e, r_Z),$$
$$w_1^3 = -\frac{1}{2}\delta r_Z(1 + 7r_Z + r_e^3 - r_e^2(5 + r_Z)$$
$$- 3r_e(7 - 2r_Z)),$$
$$w_2^3 = \delta r_Z((1 - r_Z^2) + r_Z^2 + r_e(2 + 9r_e - 6r_Z)),$$
$$w_3^3 = 0. \tag{26}$$

C. Cross sections for $\gamma e \to \nu W$

The cross sections are:

$$\sigma = \sigma_3(r_e, r_W, 0),$$
$$w_1^3 = \frac{1}{4}(r_e^3(1 - r_W) - 2r_e^2(1 - 6r_W + r_W^2)$$
$$- r_e(15 + 29r_W - 4r_W^2) + 2(4 + 5r_W + 7r_W^2)),$$
$$w_2^3 = r_e^2(1 - r_W) + r_W(r_e(9 + r_W) - 2(2 + r_W + r_W^2)),$$
$$w_3^3 = 0, \tag{27}$$
and

$$\Delta\sigma^\pm = \sigma_2(r_e, r_Z, 0),$$
$$w_1^2(+) = r_e(1 + r_e + r_W(3 - r_e)),$$
$$w_2^2(+) = -4r_W r_e,$$
$$w_3^2(+) = w_3^2(-) = 0,$$
$$w_1^2(-) = -2r_W(13 + 3r_W + r_e(5 - r_W)),$$
$$w_2^2(-) = 8r_W(1 + r_e + 3r_W), \tag{28}$$

$$\Delta\sigma = \sigma_3(r_e, r_W, 0),$$
$$w_1^3 = \frac{1}{2}r_e((1 + r_e)^2 - r_W(r_e^2 + 2r_e(2 - r_W)$$
$$- 3(1 - 4r_W))) - 4(1 - 2r_e) - r_W(5 + 7r_W),$$
$$w_2^3 = 2(-r_e^2(1 - r_W) - r_e r_W(7 - 5r_W)$$
$$+ 2r_W(2 + r_W + r_W^2),$$
$$w_3^3 = 0. \tag{29}$$

The photon-neutrino scattering formulae have been obtained assuming non-zero neutrino mass. The results for massless neutrino agree with the results previously obtained (see [38] and references therein). At the end we will complete the notation: α is electromagnetic coupling, s is square of energy in CM system and Θ_W is Weinberg's Angle.
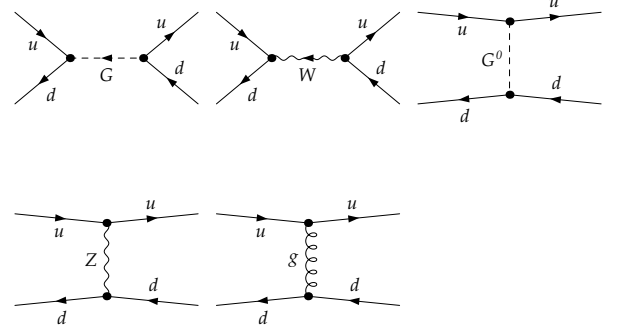

FIG. 8: The Feynmann diagrams for the subprocess: $ud \to ud$. The contribution in lowest order of perturbation theory comes from interference between terms represented by Z^0/W bosons and gluons exchanged diagrams.

X. APPENDIX B.

To estimate the high energy contributions to p.v. sum rule (5) the QCD improved parton model is used. Polarized parton distributions for proton and unpolarized photon structure have been taken from [39]. In perturbative QCD calculations a lot of subprocesses with quarks and antiquarks from photon and proton participate and in principle their contributions might be canceled. The calculations have been done in the lowest non-vanishing order in electroweak theory and QCD which gives the contribution to p.v. cross sections proportional to $\alpha\alpha_S$. Typical example is shown in fig. 8 where the diagrams for $ud \to ud$ quarks interactions are drawn. The p.v. contributions come from the interference terms between subprocesses with Z^0/W bosons and gluons exchanged between partons.

The total difference of the cross sections is a sum of contributions from all possible quark-quark and quark-antiquark interactions. The difference of the cross sections can be expressed by the following formula:

$$\Delta\sigma = \frac{32\pi\alpha}{9}\int\frac{dx_1 dx_2}{x_1 x_2}\frac{d\hat{t}}{\hat{s}^2}(F_1(x_1, x_2)(g(M_Z^2) + h(M_Z^2))$$
$$- F_2(x_1, x_2)h(M_W^2) - F_3(x_1, x_2)g(M_W^2)) \tag{30}$$

where:

$$g(x) = \frac{\alpha_S(\hat{t})}{\hat{t}}(\frac{\hat{s}^2}{\hat{u} - x} + \frac{\hat{u}^2}{\hat{s} - x}) \tag{31}$$

$$h(x) = \frac{\alpha_S(\hat{s})}{\hat{s}}\frac{\hat{u}^2}{\hat{t} - x} \tag{32}$$

and

$$F_1(x_1, x_2) = (x_2\Delta u(x_2))(x_1 u(x_1))(C_{1u}^2 - C_{2u}^2)$$
$$+ (x_2\Delta d(x_2))(x_1 d(x_1))(C_{1d}^2 - C_{2d}^2) \quad (33)$$
$$F_2(x_1, x_2) = C_{3ud}^2((x_2\Delta u(x_2))(x_1 u(x_1))$$
$$+ (x_2\Delta d(x_2))(x_1 d(x_1)))$$
$$+ C_{3us}^2(x_2\Delta u(x_2))(x_1 u(x_1)) \quad (34)$$
$$F_3(x_1, x_2) = C_{3ud}^2((x_2\Delta u(x_2))(x_1 d(x_1))$$
$$+ (x_2\Delta d(x_2))(x_1 u(x_1)))$$
$$+ C_{3us}^2(x_2\Delta u(x_2))(x_1 s(x_1)) \quad (35)$$

The polarized parton distributions in proton are denoted as $\Delta u(x_2)$, $\Delta d(x_2)$ and $\Delta s(x_2)$ for u,d ans s quarks, respectively. The photon structure (unpolarized) is described by $u(x_1)$, $d(x_1)$ and $s(x_1)$. The x_1 and x_2 are the fraction of momentum of photon and proton carried by parton (quark). The coefficients C are listed below:

$$C_{1u} = \frac{2}{3}\frac{sin(\Theta_W)}{cos(\Theta_W)}$$

$$C_{1d} = \frac{1}{3}\frac{sin(\Theta_W)}{cos(\Theta_W)}$$

$$C_{2u} = \frac{3 - 4sin^2(\Theta_W)}{6sin(\Theta_W)cos(\Theta_W)}$$

$$C_{2d} = \frac{3 - 2sin^2(\Theta_W)}{6sin(\Theta_W)cos(\Theta_W)}$$

$$C_{3ud} = \frac{CKM_{11}}{\sqrt{(2)}sin(\Theta_W)}$$

$$C_{3ud} = \frac{CKM_{12}}{\sqrt{(2)}sin(\Theta_W)} \quad (36)$$

The integration is performed over kinematically allowed ranges and $\hat{s} = x_1 x_2 s$. The perturbative scale has been fix as $Q_0^2 = 1 GeV^2$ and $\hat{s} > 4Q_0^2$. CKM_{11} and CKM_{12} are Cabbibo-Kobayashi-Maskawa quark-mixing matrix elements.

It is worth to note that the color preservation forbids some subprocesses to contribute to cross sections. The interference terms for right-handed cross sections in the case of different flavours of quarks (e.g. ud,us, etc) give vanishing contributions when the summation over colors is performed. The numerical results for the difference of the cross sections as a function of CMS energy $\sqrt{s}$ are shown in fig. 9.

The differences of the cross sections are sizeable (up to 60 pb) and rising up (in absolute value sense) as it is shown in fig. 9. It indicates that for the unpolarized photons case the p.v. sum rule is violated not only in the case of point-like lepton targets but also for polarized proton ones. The small momenta partons are responsible for the rising of the absolute value of difference of the cross sections.
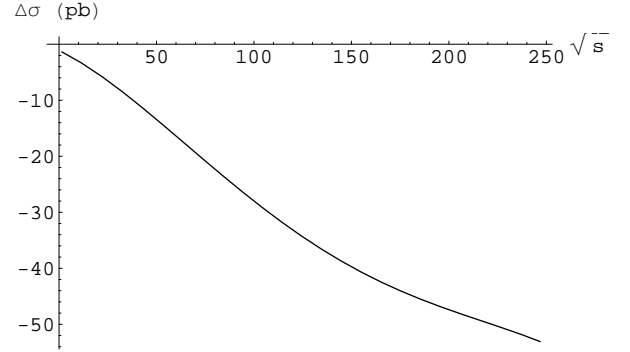
FIG. 9: The CMS energy dependence of the difference of the cross sections for unpolarized photons scattered off polarized protons, calculated in perturbative QCD. The parton structure of photon and proton are taken from [39].

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